

# Multiple scattering effects in quasi free scattering from halo nuclei: a test to Distorted Wave Impulse Approximation

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Full Faddeev-type calculations are performed for  $^{11}\text{Be}$  breakup on proton target at 38.4, 100, and 200 MeV/u incident energies. The convergence of the multiple scattering expansion is investigated. The results are compared with those of other frameworks like Distorted Wave Impulse Approximation that are based on an incomplete and truncated multiple scattering expansion.

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## I. INTRODUCTION

Interest in quasi free breakup of a Halo nucleus stems mainly from the structure information about the wave function of the struck valence neutron. Qualitatively, the quasi free scattering is a process [1, 2] in which a particle knocks a nucleon out of the nucleus (A) and no further strong interaction occurs between the  $(A_{n-1})$  nucleus and the two outgoing particles.

Assuming that the nucleus (A) can be well described by an inert  $(A_{n-1})$  core and a valence neutron, this reaction can be studied using the few-body Faddeev or equivalent Alt, Grassberger, and Sandhas (AGS) scattering framework [3, 4, 5] which can be viewed as a multiple scattering series in terms of the transition amplitudes for each interacting pair. Both equations are consistent with the corresponding Schrödinger equation and therefore provide an exact description of the quantum three-body problem but are more suitable for the numerical solution. The Faddeev equations are formulated for the components of the wave function, while the AGS equations are integral equations for the transition operators that lead more directly to the scattering observables.

Traditionally this reaction has been analysed using the Distorted Wave Impulse Approximation (DWIA) scattering framework [6]. In this approach one assumes that the incoming particle collides with the struck nucleon as if it was free, and therefore the cross section for the quasi free scattering is given by the product of the transition amplitude between the two colliding particles modulated by the wave function of the nucleon inside the nucleus. Furthermore the relative motion of the particles in the entrance and exit channels is described by distorted waves. This approximation leads, in each order, to an incomplete and truncated multiple scattering expansion.

In order to obtain accurate information about the wave function, one needs to investigate the effect of higher order rescattering between the two colliding particles and the  $(A_{n-1})$  nucleus in the calculated knockout observables. In addition, the contribution of the neglected terms in each order needs to be assessed. This is the

aim of the present work.

We consider here the quasi free scattering of the A nucleus by a proton target leading to the breakup reaction  $p(A, A_{n-1}n)p$ . We take as an example the breakup of  $^{11}\text{Be}$  halo into a proton target at 38.4, 100, and 200 MeV/u.

Recently the elastic scattering of  $^{11}\text{Be}$  from a proton target was analysed using the Faddeev/AGS multiple scattering approach [7]. It was shown that even at intermediate energies of 200 MeV/u one needs to take into account 3rd order multiple scattering contributions that are not included in the Glauber scattering framework [8].

In this work we briefly review the Faddeev/AGS multiple scattering framework in section II. In section III we summarize the DWIA formalism and identify the terms of the corresponding multiple scattering expansion. In section IV we show the calculated fivefold breakup cross section using the Faddeev/AGS equations and compare with the results of the DWIA type.

## II. THE FADDEEV/AGS EQUATIONS

In this section, for completeness we briefly describe the Faddeev/AGS multiple scattering framework which treats all open channels in the same footing. Let us consider 3 particles (1,2,3) interacting by means of two-body potentials. In this section we use the odd man out notation appropriate for 3-body problems which means, for example, that the interaction between the pair (2,3) is denoted as  $v_1$ . We assume that the system is non-relativistic and we write its total Hamiltonian as

$$H = H_0 + \sum_{\gamma} v_{\gamma}, \quad (1)$$

with the kinetic energy operator  $H_0$  and the interaction  $v_{\gamma}$  for the pair  $\gamma$ . The Hamiltonian can be rewritten as

$$H = H_{\alpha} + V^{\alpha}, \quad (2)$$

where  $H_{\alpha}$  is the Hamiltonian for channel  $\alpha$

$$H_{\alpha} = H_0 + v_{\alpha}, \quad (3)$$

and  $V^\alpha$  represents the sum of interactions external to partition  $\alpha$

$$V^\alpha = \sum_{\gamma \neq \alpha} v_\gamma. \quad (4)$$

The  $\alpha = 0$  partition corresponds to three free particles in the continuum where  $V^0$  is the sum of all pair interactions. The total transition amplitude which describes the scattering from the initial state  $\alpha$  to the final state  $\beta$  is given in the post form (see Refs. [5, 7] for a derivation)

$$T_+^{\beta\alpha} = V^\beta + \sum_{\gamma \neq \alpha} T_+^{\beta\gamma} G_0 t_\gamma \quad (5)$$

with the transition amplitude

$$t_\gamma = v_\gamma + v_\gamma G_0 t_\gamma \quad (6)$$

and the free resolvent

$$G_0 = (E + i0 - H_0)^{-1}, \quad (7)$$

where  $E$  is total energy of the three-particle system. Equivalently in the prior form we write

$$T_-^{\beta\alpha} = V^\alpha + \sum_{\gamma \neq \beta} t_\gamma G_0 T_-^{\gamma\alpha}. \quad (8)$$

One then defines the operators  $U^{\beta\alpha}$  which are equivalent to the transition amplitudes on the energy shell, such that

$$\begin{aligned} U^{\beta\alpha} &= \bar{\delta}_{\beta\alpha} G_\alpha^{-1} + T_+^{\beta\alpha} \\ &= \bar{\delta}_{\beta\alpha} G_\beta^{-1} + T_-^{\beta\alpha}, \end{aligned} \quad (9)$$

where  $\bar{\delta}_{\beta\alpha} = 1 - \delta_{\beta\alpha}$ . From Eqs. (5-8)

$$U^{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\gamma} U^{\beta\gamma} G_0 t_\gamma \bar{\delta}_{\gamma\alpha}, \quad (10)$$

or

$$U^{\beta\alpha} = \bar{\delta}_{\beta\alpha} G_0^{-1} + \sum_{\gamma} \bar{\delta}_{\beta\gamma} t_\gamma G_0 U^{\gamma\alpha}. \quad (11)$$

These are the well known three-body AGS equations [4] which for breakup ( $\beta = 0$  in the final state) become

$$U^{0\alpha} = G_0^{-1} + \sum_{\gamma} t_\gamma G_0 U^{\gamma\alpha}, \quad (12)$$

where  $U^{\gamma\alpha}$  is obtained from the solution of Eq. (11) with  $\alpha, \beta, \gamma = (1, 2, 3)$ . The scattering amplitudes are the matrix elements of  $U^{\beta\alpha}$  calculated between initial and final states that are eigenstates of the corresponding channel Hamiltonian  $H_\alpha$  ( $H_\beta$ ) with the same energy eigenvalue. For elastic scattering and transfer those states are made up by the bound state wave function for pair  $\alpha$  ( $\beta$ ) times a relative plane wave between particle  $\alpha$  ( $\beta$ ) and pair  $\alpha$  ( $\beta$ ). For breakup the final state is a product of two plane

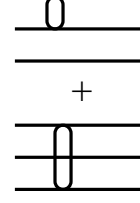


FIG. 1: Single scattering diagrams for breakup in the Faddeev scattering framework.

waves corresponding to the relative motion of three free particles that may be expressed in any of the relative Jacobi variables. In the latter case the contribution of the  $G_0^{-1}$  term is zero.

The solution of the Faddeev/AGS equations can be found by iteration leading to

$$\begin{aligned} U^{0\alpha} &= \sum_{\gamma} t_\gamma \bar{\delta}_{\gamma\alpha} + \sum_{\gamma} t_\gamma \sum_{\xi} G_0 \bar{\delta}_{\gamma\xi} t_\xi \bar{\delta}_{\xi\alpha} \\ &+ \sum_{\gamma} t_\gamma \sum_{\xi} G_0 \bar{\delta}_{\gamma\xi} t_\xi \sum_{\eta} G_0 \bar{\delta}_{\xi\eta} t_\eta \bar{\delta}_{\eta\alpha} \\ &+ \dots, \end{aligned} \quad (13)$$

where the series is summed up by the Padé method [9]. The successive terms of this series can be considered as first order (single scattering), second order (double scattering) and so on in the transition operators. The breakup series up to third order is represented diagrammatically in Figs. 1 - 3 where the upper particle is taken as particle 1 scattering from the bound state of the pair (23).

In our calculations Eqs. (11 - 13) are solved exactly after partial wave decomposition and discretization of all momentum variables. Since, in addition to the nuclear interaction between all three pairs, we include the Coulomb interaction between the proton and  $^{10}\text{Be}$ , we follow the technical developments implemented in Refs. [10, 11] for proton-deuteron and  $\alpha$ -deuteron elastic scattering and breakup that were also used in Ref. [7] to study p- $^{11}\text{Be}$  elastic scattering. The treatment of the Coulomb interaction in the framework of three-body Faddeev/AGS equations is based on the method of screening of the Coulomb interaction plus renormalization [10, 11]; it leads to fully converged results for the on-shell elastic, transfer and breakup observables that are independent of the choice of screening radius used in the calculations, as long as it is sufficiently large ( $R \geq 10$  fm for the observables considered in the present work).

### III. THE DWIA/PWIA

Let us consider the reaction  $A(a, ab)B$  where an incident particle  $a$  knocks out a nucleon or a bound cluster  $b$  in the target nucleus  $A$  resulting in three particles  $(a, b, B)$  in the final state. This reaction has been analyzed within the DWIA as described for example in the

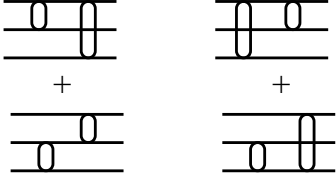


FIG. 2: Double scattering diagrams for breakup in the Faddeev scattering framework.

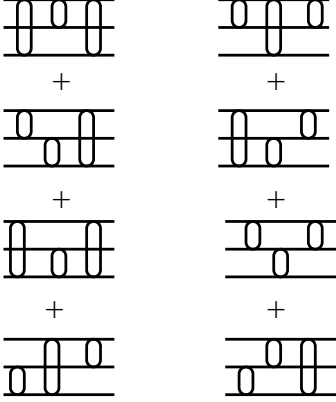


FIG. 3: Triple scattering diagrams for breakup in the Faddeev scattering framework.

work of Chant and Roos [6]. This is an approximate formalism and its validity in the application to radioactive beam studies needs to be assessed. In the work of reference [6] it is assumed that the projectile strikes the ejected particle freely and that the transition amplitude is given by

$$T_{AB} = \langle \eta_{Bab}^{(-)} | t_{ab} | \phi_{Bb} \eta_{aA}^{(+)} \rangle, \quad (14)$$

where  $\phi_{Bb}$  is the  $(Bb)$  bound state wave function, and  $\eta_{aA}^{(+)}$  and  $\eta_{Bab}^{(-)}$  describe the relative motion of the particles in the entrance and exit channels, respectively, neglecting the interaction  $V_{ab}$ . Therefore in the entrance channel  $\eta_{aA}^{(+)}$  satisfies

$$(T_{aA} + V_{aA} - V_{ab} - \epsilon_{aA}) \eta_{aA}^{(+)} = 0, \quad (15)$$

where  $\epsilon_{aA}$  is the relative kinetic energy and  $T_{aA}$  the relative kinetic energy operator. In standard DWIA calculations one takes  $V_{aA} - V_{ab} \sim V_{aB}(\vec{r}_{aA})$ . For the exit channel one assumes as a good approximation to write the 3-body wave function  $\eta_{Bab}^{(-)}$  as a factorized product

$$\eta_{Bab}^{(-)} \sim \eta_{aB}^{(-)} \eta_{bB}^{(-)}, \quad (16)$$

where

$$(T_{aB} + V_{aB} - \epsilon_{aB}) \eta_{aB}^{(+)} = 0, \quad (17)$$

and

$$(T_{bB} + V_{bB} - \epsilon_{bB}) \eta_{bB}^{(+)} = 0. \quad (18)$$

The potentials  $V_{aB}$  and  $V_{bB}$  are taken to be the optical potentials which describe the  $a + B$  and  $b + B$  scattering at energies  $\epsilon_{aB}$  and  $\epsilon_{bB}$  respectively. Therefore one writes

$$T_{AB} \sim \langle \eta_{aB}^{(-)} \eta_{bB}^{(-)} | t_{ab} | \phi_{Bb} \eta_{aA}^{(+)} \rangle. \quad (19)$$

If we now introduce plane waves in the exit channel

$$T_{AB} \sim \langle \chi_{aB} \chi_{bB} | (1 + t_{aB} G_0) (1 + t_{bB} G_0) t_{ab} | \phi_{Bb} \eta_{aA}^{(+)} \rangle, \quad (20)$$

we then obtain the truncated multiple scattering series

$$t_{ab} + t_{aB} G_0 t_{ab} + t_{bB} G_0 t_{ab} + t_{aB} G_0 t_{bB} G_0 t_{ab}. \quad (21)$$

The first order term is the single scattering represented diagrammatically in Fig. 4. This is the only term retained in Plane Wave Impulse Approximation (PWIA) calculations. The second and third order terms in Eq. (21) represented in Fig. 5 are due to distortion in the exit channel. The relative importance of these terms to the scattering can be assessed by performing Faddeev multiple scattering calculations.

The distortion in the entrance channel is more difficult to bridge with few-body calculations. In order to estimate this distortion in the case of a light emitted particle one could say for instance, that

$$\eta_{aA}^{(+)} \sim (1 + G_0 t_{aB}) \chi_{aA}. \quad (22)$$

This leads to additional multiple scattering contributions

$$t_{ab} G_0 t_{aB} + t_{aB} G_0 t_{ab} G_0 t_{aB} + t_{bB} G_0 t_{ab} G_0 t_{aB} + \mathcal{O}(t^4). \quad (23)$$

These terms are represented in Fig. 6. We note that the distortion on the entrance and exit channels contribute to a truncated and incomplete multiple scattering series at each order.

We concentrate on the study of the multiple scattering expansion. In addition, we want to investigate if the single scattering term becomes the dominant contribution as the energy increases. It was shown in the work of Chant and Roos [6] that, contrary to what is expected, the DWIA results do not converge to the PWIA results as the projectile energy increases. The importance of including the distortion in comparison with the full Faddeev multiple scattering series is also estimated in the present work. We refer as DWIA-full a multiple scattering calculation that includes the multiple scattering terms in Eqs. (21) and (23). The calculations based on these equations follow exactly the same procedure as explained before for the solution of the Faddeev/AGS equations (11) through (13), both in terms of initial and final states as well as treatment of Coulomb, so that one may obtain a consistent comparison between different terms in the multiple scattering representation of DWIA and Faddeev/AGS amplitudes.

We note that further approximations are usually made in standard applications of the DWIA when evaluating the transition amplitude, namely:

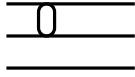


FIG. 4: Single scattering diagram in the DWIA scattering framework.

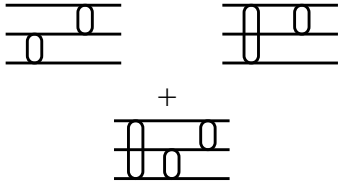


FIG. 5: Diagrams due to the distortion in the exit channel in the DWIA scattering framework.

- The potential approximation in the entrance channel  $V_{aA} - V_{ab} \sim V_{aB}(\vec{r}_{aA})$ .
- The factorization approximation, which is only exact in PWIA.
- On-shell approximation of the transition amplitude.

We refer to this as DWIA-standard. An attempt to estimate the effect of such approximations was done for example in the work of Ref. [12] for (p,2p) reactions. The Faddeev calculations and the DWIA-full do not make use of such approximations in the summation of the multiple scattering expansion, and therefore we are not addressing here the validity of such approximations which are responsible for additional shortcomings of DWIA-standard vis-a-vis the exact calculations.

#### IV. RESULTS

We now proceed to solve Faddeev/AGS equations (11 - 13) and to calculate the observables for the proton- $^{11}\text{Be}$  breakup. As mentioned above the treatment of the Coulomb interaction, as well as the calculational technique, is taken over from Refs. [10, 11].

First we describe the pair interactions p-n, p- $^{10}\text{Be}$  and n- $^{10}\text{Be}$ . For p-n we take the realistic nucleon-nucleon CD Bonn potential [13]. The interaction between the valence neutron and the  $^{10}\text{Be}$  core is assumed to be L-dependent

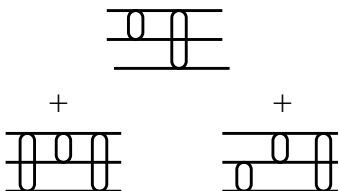


FIG. 6: Diagrams due to the distortion in the incoming channel in the DWIA scattering framework.

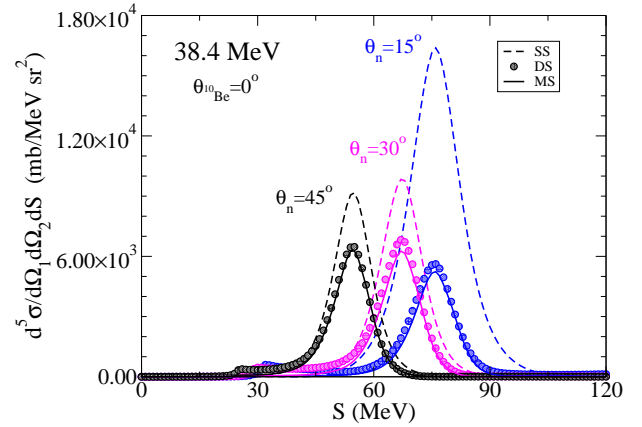


FIG. 7: (Color online) Cross section for the breakup  $^{11}\text{Be}(p,pn)$  at 38.4 MeV.

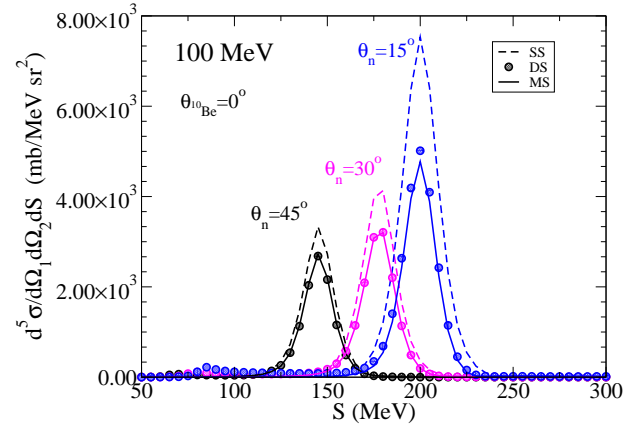


FIG. 8: (Color online) Cross section for the breakup  $^{11}\text{Be}(p,pn)$  at 100 MeV.

as described in [7]. For the potential between the proton and  $^{10}\text{Be}$  core we use a phenomenological optical model with parameters taken from the Watson global optical potential parametrization [7, 15]. The energy dependent parameters of the optical potential are taken at the proton laboratory energy of the p- $^{11}\text{Be}$  reaction in inverse kinematics.

In the solution of the Faddeev equations we include n-p partial waves up to relative total angular momentum  $\ell_{np} \leq 8$ , n- $^{10}\text{Be}$  up to  $\ell \leq 3$ , and p- $^{10}\text{Be}$  up to  $L \leq 10$ . Three-body total angular momentum is included up to 100.

As a first result we show in Figs. 7–9 the fivefold differential breakup cross section  $d^5\sigma/d\Omega_1 d\Omega_2 dS$  versus the arclength  $S$  at  $E_{\text{lab}}/A = 38.4$  MeV, 100 MeV, and 200 MeV respectively. The kinematical configurations are characterized by the polar and azimuthal angles  $(\theta_i, \phi_i)$  of the two detected particles labeled 1 and 2 in Fig. 10. We assume those particles to be the  $^{10}\text{Be}$  core (1) and the neutron (2). We take three quasi free scattering configurations: (conf 1) with  $(\theta_1, \phi_1) = (0^\circ, 0^\circ)$  and

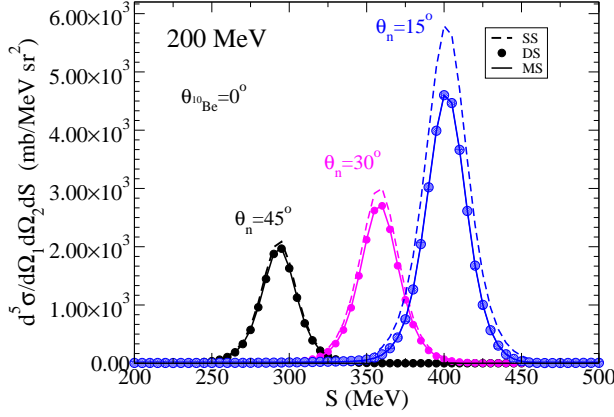


FIG. 9: (Color online) Cross section for the breakup  $^{11}\text{Be}(p,pn)$  at 200 MeV.

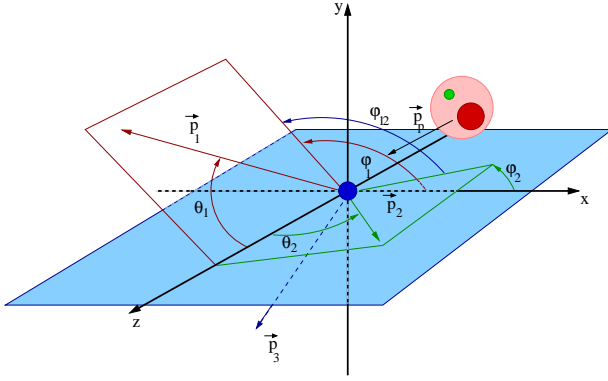


FIG. 10: (Color online) Kinematic angles for breakup.

$(\theta_2, \phi_2) = (45^\circ, 180^\circ)$ ; (conf 2) with  $(\theta_1, \phi_1) = (0^\circ, 0^\circ)$  and  $(\theta_2, \phi_2) = (30^\circ, 180^\circ)$ ; (conf 3) with  $(\theta_1, \phi_1) = (0^\circ, 0^\circ)$  and  $(\theta_2, \phi_2) = (15^\circ, 180^\circ)$ . The arclength  $S$  is related to the energies  $E_i$  of the two detected particles in the standard way [14] as  $S = \int_0^S dS$  with  $dS = (dE_1^2 + dE_2^2)^{1/2}$ . In each configuration, the solid lines represent the full multiple scattering (MS) results obtained from the solution of Faddeev/AGS equations. The single scattering approximation (SS) represented by the dashed lines is calculated taking into account the proton scattering from the struck nucleon and the scattering from the valence core as represented by diagrams in Fig. 1. As follows from Fig. 7 the SS approximation overestimates the exact result in any configuration at 38.4 MeV/u. At 200 MeV/u, the SS already provides a good representation of the full calculation in the case of conf 1, but increasingly overestimates MS in conf 2 or conf 3 where the neutron is emitted at smaller angles. Thus, contrary to what is predicted by Chant and Roos [6], if the neutron is not detected at very forward angles, the single scattering approaches the full calculation as the beam energy is increased. The double scattering (DS) approximation, represented by the circles, and calculated taking into account the diagrams of Figs. 1 and 2, re-

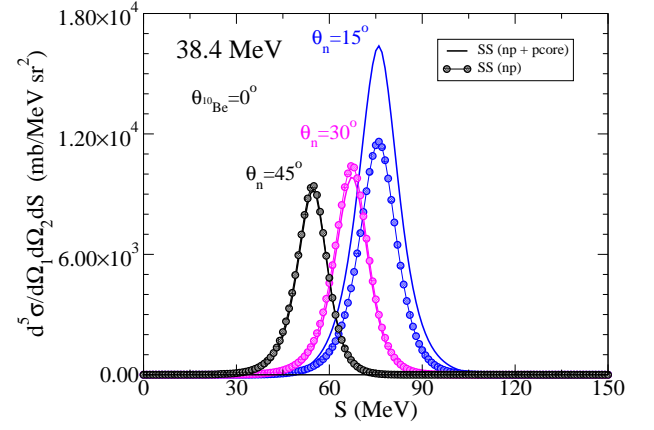


FIG. 11: (Color online) Cross section for the breakup  $^{11}\text{Be}(p,pn)$  at 38.4 MeV.

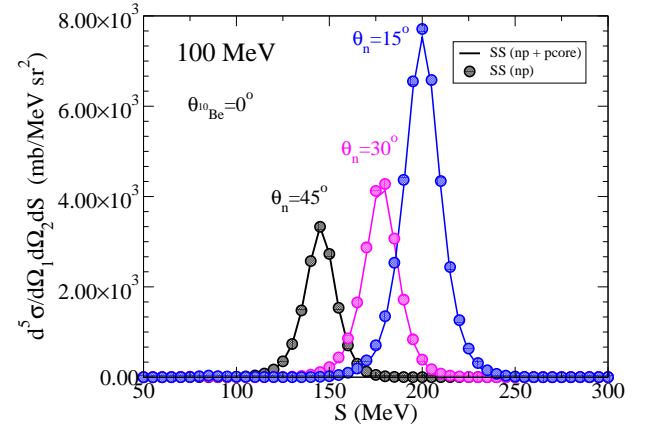


FIG. 12: (Color online) Cross section for the breakup  $^{11}\text{Be}(p,pn)$  at 100 MeV.

produces well the full calculation at 200 MeV/u energy. Therefore the Faddeev/AGS multiple scattering series for breakup has converged at second order level at this energy.

Now we compare Faddeev/AGS and DWIA-full results for the breakup observables. We begin by analyzing the single scattering term in Figs. 11–13. The breakup cross section calculated taking into account both proton-neutron and proton-core contributions is represented by the solid line. The circles correspond to the PWIA, i.e., without the proton-core scattering contribution. At 38.4 MeV/u this contribution is very small in the configurations 1 and 2, but is sizable when the neutron is emitted in the forward region. Therefore, even at the single scattering level, calculations become inaccurate in some configurations at low energies. At high energies the proton-core scattering contribution is negligible in all configurations.

Since, as shown in Figs. 7–9, the DS results are very close to the full MS results, we compare next in Figs. 14–16 the predictions of Faddeev/AGS and the DWIA-full

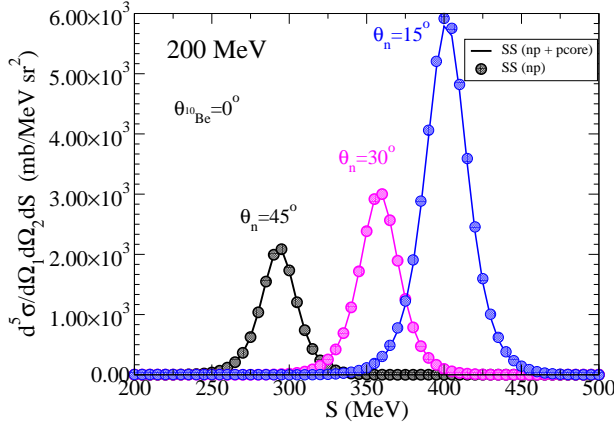


FIG. 13: (Color online) Cross section for the breakup  $^{11}\text{Be}(p,pn)$  at 200 MeV.

approach including in both calculations the corresponding multiple scattering terms up to second order. As explained before the DWIA-full approach provides an incomplete series in all orders, though in first order the effects of the missing term are negligible at high energies as shown in Figs. 11–13.

The Faddeev/AGS (solid line) results include all the single and double scattering terms shown in Figs 1–2. The DWIA-full assumes that the contribution of the scattering between the proton and the neutron is dominant and therefore the single scattering contribution due to the scattering of the proton from the  $^{10}\text{Be}$  core is neglected. The circles include the first and second order diagrams originated from the distortions in the exit channel represented in Fig. 5, and the dashed curve includes, in addition, the second order diagrams originated from the distortion in the incoming channel represented in Fig. 6. The DWIA-full results to second order (dashed line) provide a poor approximation to the Faddeev/AGS at 38.4 MeV in any configuration. As the energy increases DWIA-full approaches the exact result. However one should keep in mind that DWIA-standard calculations assume further approximations as discussed in Sect. III.

In Figs. 9 and 11–13 we show that in the high energy regime PWIA approaches the exact result at least in some suitable breakup configurations. Therefore, in this energy regime, in order to get a reliable prediction of the breakup observables in quasi free scattering, we advocate that a better approach is to work with the PWIA scattering framework within suitably chosen configurations rather than in DWIA where uncontrollable approximations are made. On the contrary, in PWIA the factorization is exact and the transition amplitude for the scattering between the incident and knocked particle is on the energy shell in the weak binding limit of the knocked particle.

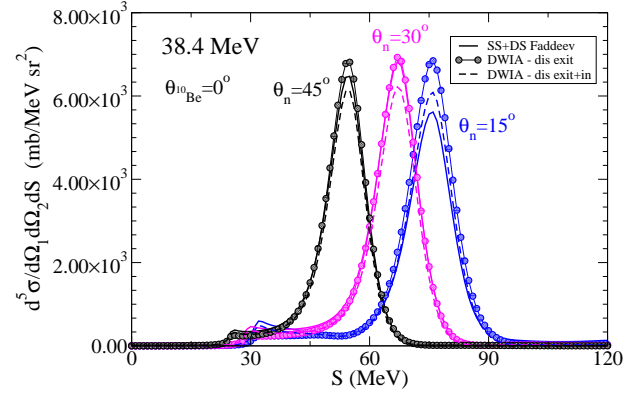


FIG. 14: (Color online) Cross section for the breakup  $^{11}\text{Be}(p,pn)$  at 38.4 MeV.

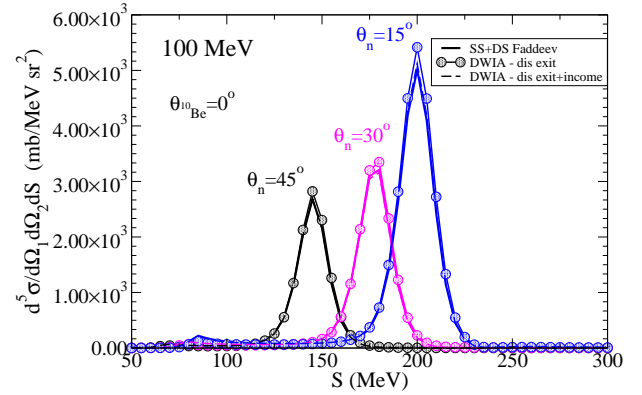


FIG. 15: (Color online) Cross section for the breakup  $^{11}\text{Be}(p,pn)$  at 100 MeV.

## V. CONCLUSIONS

We have calculated the breakup cross section for  $^{11}\text{Be}$  breakup from protons at 38.4, 100 and 200 MeV/u. Quasi free scattering conditions are considered in different configurations for the emitted neutron. The Faddeev/AGS

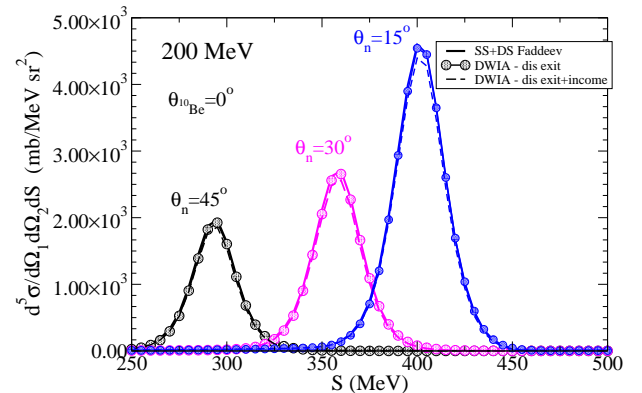


FIG. 16: (Color online) Cross section for the breakup  $^{11}\text{Be}(p,pn)$  at 200 MeV.

formalism is used to show that at these energies the multiple scattering expansion has converged at the second order. We have also shown that at high incident energies the single scattering (PWIA) approaches the full calculation in some suitable configurations, namely when the neutron is not emitted at forward angles. We have also shown that the DWIA-full scattering framework provides a poor approximation of the Faddeev/AGS formalism at low energies, but approaches it at high energies. However since standard applications make further use of un-

controlable approximations such as the factorization and on-shell approximations, one advocates that at these high energies it is advisable to use the exact approach or instead the PWIA in suitable configurations.

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